Opinion Games

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Questions and a New Model

Motivating Questions: How do opinions change? How do social networks shape opinion? Is there a connection between the ground being lost by moderates and the growing gap between the extremes in politics? Has the way in which the Internet mediates influence fundamentally changed the way in which opinions evolve?

Simplified quantitative models of the formation and evolution of opinions on a topic (or several topics) are referred to as *opinion games* and have been an interest to scientists for at least 7 decades [1–3, 5]. We have created a new, flexible opinion game model [4][1]. The added complexity in our family of models has already generated some interesting preliminary results. Here are some examples:

Known Games are Special Cases Our model system, based on variational ideas, is rich enough that most of the well-known old models can be embedded in our model.

Moderates Moderate Clusters of moderate opinions can keep the more extreme clusters from becoming even more polarized.

Coupled Topics can Increase Polarization We found examples of opinion systems where the addition of coupling between topics increased polarization.

Model Details

The basic ingredients in our model are:

Agents and Interaction Network: There are agents that interact and adjust opinions on topics as a result of those interactions. Each agent (or network node) interacts with all the other agents it is connected to as determined by the interaction graph or network.

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[1] The paper is accessible here https://arxiv.org/abs/1607.06806

Opinion Space $\mathcal{O}$: represented by numbers in the interval $\mathcal{O} = [0, 1]$, 0 meaning absolute negative opinion, and 1 being absolute positive opinion about a topic (More generally opinions live in hyper-cubes).

Potentials ψ: These potentials are the interaction energies which the dynamics seek to minimizes. Because they are not necessarily symmetric (agents A and B can use different interaction potentials to respond to the same interaction) the resulting evolution need not be a simple gradient evolution.

Topic Space and Coupling: We can have a discrete set or continuum of topics we evolve, permitting opinions on different topics to be coupled using one of two different coupling models.

Dynamics: The dynamics can be continuous and synchronously evolving – this is the classical dynamical systems approach, discrete and deterministic in interaction order, and discrete with random interaction order. In either case, the change is dictated by the gradients of the interaction potentials.

The flexibility of this variational approach allows for a very rich set of behaviors, mediated by a rich set of potentials. In the paper in which we introduced this new approach [4], we focused on potentials which are functions of opinion differences. Even in that case, we found interesting, emergent behavior.

We studied both the continuous, synchronous version and discrete, stochastic, sequential version of the game. Opinion at the nodes (agents) was changed so as to reduce the interaction energies. The discrete update rule:

$$o_i^{(t+1)} = o_i^{(t)} - \frac{\alpha}{2}\, \psi'(d_{ij}^{(t)})\, \frac{o_i^{(t)} - o_j^{(t)}}{|o_i^{(t)} - o_j^{(t)}|} \text{ and } o_j^{(t+1)} = o_j^{(t)} + \frac{\alpha}{2} \psi'(d_{ij}^{(t)})\, \frac{o_i^{(t)} - o_j^{(t)}}{|o_i^{(t)} - o_j^{(t)}|} \tag{1}$$

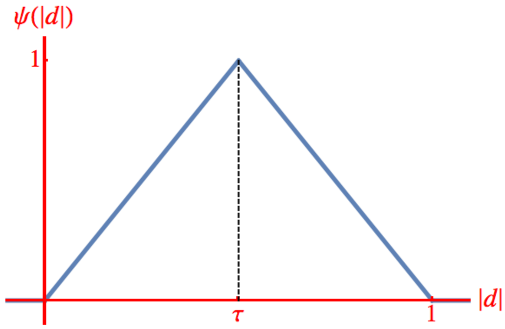
is used to update the opinions of each pair of interacting nodes. (Note that we can use different potentials for each part in an interaction, though in this particular case, we are using the same potential for both.) The differential equation describing the continuous version of the game is given by:

$$\dot{o}_i = -\frac{\alpha}{2}\, \psi'(d_{ij}^{(t)})\, \frac{o_i^{(t)} - o_j^{(t)}}{|o_i^{(t)} - o_j^{(t)}|} \text{ and } \dot{o}_j = \frac{\alpha}{2} \psi'(d_{ij}^{(t)})\, \frac{o_i^{(t)} - o_j^{(t)}}{|o_i^{(t)} - o_j^{(t)}|} \tag{2}$$

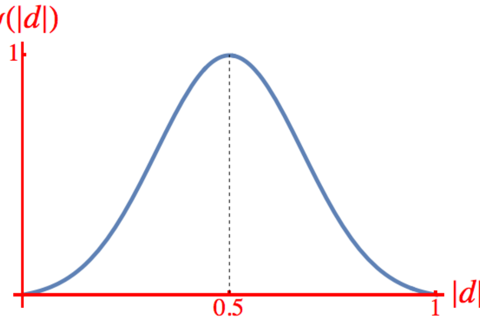
Using potentials that were tent-like or bell-like, we found that not only did we get equilibrium states in which all the agents agreed (Consensus) or where there were two clusters, one a 0 and one at 1 (Polarization), we also found emergent states that were niether complete polarization nor consensus. In essence, the moderate cluster (close to 0.5) prevented the somewhat negative opinion cluster (close to 0.15) and the somewhat positive cluster (near 0.85) from pushing each other to the extremes of complete polarization. **In other words, moderates moderated.** Figure (1) shows the potentials that were used for some of the experiments.

Finding these equilibria is not hard. If potential is a smooth bell-shaped function, we need to have a solution for the following, since ψ' depends on the distances:

$$\psi'(d_{12}) = \psi'(d_{23}) = -\psi'(d_{13}) \tag{3}$$



(a) Tent Potential



(b) Bell-Shaped Potential

Figure 1: potential function examples

In the case that ψ is symmetric and ψ' changes monotonically on $[0.25,\ 0.5]$, then Eq. (3) means we have to have $d_{12} = d = d_{23}$ where $d \in [0.25,\ 0.5]$ and $2d > 0.5$.

For example, let the bell-shaped potential be the Gaussian function with $\mu = 0.5$ and $\sigma = \frac{1}{\sqrt{2}}$, then, $\psi'(x) = -\frac{2(x-0.5)}{\sqrt{\pi}} e^{-(x-0.5)^2}$ and we must have a solution for:

$$-\frac{2(d-0.5)}{\sqrt{\pi}} e^{-(d-0.5)^2} = \frac{2(2d-0.5)}{\sqrt{\pi}} e^{-(2d-0.5)^2}$$

which yields to:

$$e^{3d^2-d} = \frac{2d-0.5}{0.5-d} \tag{4}$$

which has a solution. See Figure (2).

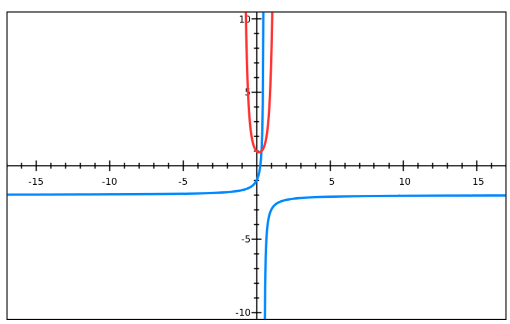


Figure 2: This figure shows that Eq.(4) has a solution. Left and right sides of the equation intersect.

Our model allows for coupling between topics in a system with multiple topics. In real life topics, are correlated to each other in different ways. For example one's opinion of healthy food and soda and diabetes are not normally be unrelated. And in a single interaction two people might not talk about all the correlated topics. Therefore, In our model we introduce a two coupling methods by which opinions on different topics are linked.

Though we have yet to study coupling in more than a cursory way, **early experiments found a system in which coupling encouraged polarization, though not without conflict.**

Code

We have developed code anyone can download and play with. The code is appropriate mostly for those with some coding experience in that we are not actively supporting the code at this time. Here is a link to the code on Github: https://github.com/HNoorazar/PyOpinionGame.

References

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